Crumpled states of a wire in a two-dimensional cavity with pins

M. A. F. Gomes,[1] V. P. Brito,[2] M. S. Araújo,[2] and C. C. Donato[3]
[1]*Departamento de Física, Universidade Federal de Pernambuco, 50670-901 Recife, PE, Brasil*
[2]*Departamento de Física, Universidade Federal do Piauí, 64049-550 Teresina, PI, Brasil*
[3]*Instituto de Física, Universidade de Brasília, 70910-900 Brasília, DF, Brasil*
(Received 8 May 2009; revised manuscript received 29 January 2010; published 26 March 2010)

In this paper, we report an extensive experimental study of the configurations of a plastic wire injected into a two-dimensional planar cavity populated with fixed pins. The wire is not allowed to cross any pin, but it can move in a wormlike manner within the cavity until to become jammed in a crumpled state. The jammed packing fraction depends heavily on the topology of the cavity, which depends on the number of pins. The experiment reveals nontrivial entanglement effects and scaling laws which are largely independent of the details of the distribution of pins, the symmetry of the cavity or the type of the wire. A mean-field model for the process is presented and analogies with some basic aspects of statistical thermodynamics are discussed.

DOI: 10.1103/PhysRevE.81.031127 PACS number(s): 05.70.Np, 46.70.Hg, 62.20.F−, 68.65.−k

I. INTRODUCTION

Physics in two dimensions has proven to be of great importance to technology as well as to pure science. Experiments done in the few last decades have revealed a two-dimensional (2D) world of rich and intriguing phenomena in its own right [1–5]. Within the context of 2D systems, the basic problem of the nonthermal forced packing of a single piece of wire injected into a 2D simply connected planar cavity (i.e., a cavity whose interior is free of holes or excluded domains) has aroused a crescent interest only in the last few years. Originally, several aspects of this packing problem were studied in connection with crumpling and scaling laws [6,7]. In particular, in [6] it was developed a two-parameter hierarchical model that gives a good description of the basic statistical functions associated with the rigid, noncompact, heterogeneous 2D structures of crumpled wires (2D CWs) in the elastic limit. Afterward, many physical problems associated with the low-dimensional packing of wires were analyzed, as, e.g., the anomalous diffusion on a 2D CW structure [8], the interplay between elasticity and geometry in rods and strings [9–11], the condensation of elastic energy in 2D CW [12], as well as universality aspects in the multiple coiling of elastic sheets and fibers [13], the packing of DNA [14,15], phase diagrams and numerical simulations [16,17], plastic deformations [18], topological crossovers in crumpled systems [19], and conformal maps and 2D gravity [20]. In all these studies the cavities used to accommodate elastic, elastoplastic and, in a minor extent, plastic wires were simply connected. Differently, in the present paper we study experimentally in detail the structures obtained when a plastic wire of circular section is injected into 2D (or quasi-2D) planar cavities of fixed size but different topologies. In order to vary systematically the topology we have introduced within the cavity regular distributions of cylindrical pins localized on the sites of square lattices. The progressive addition of pins in the interior of the cavity gives origin to packing of wires presenting different morphologies and obeying scaling laws. Although we are mainly interested in general in geometric and statistical properties associated with the patterns of plastic wires in cavities with pins, the subject examined in the present study can be of interest in a number of important fields as, for instance, self-avoiding walks and polymer configurations on the plane. As an illustration, the phenomenon reported here is reminiscent of the reptation phenomenon of a single unbranched polymeric chain trapped in a network of obstacles. This important problem in polymer science and technology gives origin, in fact, to nontrivial entanglement effects [21]. Other potential implications of our study are in the packing of DNA segments constrained to wrap around histone cores [22], as well as in the electrohydrodynamics of DNA in confined environments [23].

This paper is organized as follows. In Sec. II the experimental details are described; in Sec. III a discussion of the results is presented, and in Sec. IV a mean-field model is introduced in order to understand some of our findings in terms of statistical thermodynamics ideas. A brief summary of the paper and its main conclusions are given in Sec. V.

II. EXPERIMENTAL DETAILS

In our experiments we have used confining cells whose interior cavities were of square shape with an edge L_0 of 150 mm. The bottom parts of the cavities were made of a single piece of wood with height 20 mm which facilitated the implantation of cylindrical pins of steel along the sites of regular square lattices. Nine cavities with the number of pins n =1, 4, 9, 16, 25, 36, 64, 144, and 196 were used. The position of the pins is defined by dividing the area L_0^2 of the cavity in n equal nonoverlapping and contiguous squares of area L_0^2/n. Each pin was placed in the geometric center of those squares, and so the nearest-neighbor pin-pin distance was $l=L_0/n^{1/2}$. In our experiments the diameter and height of the pins, and the height of the cavity are equal to the diameter of the wire, $\zeta=1.5$ mm, in such a way that only structures with a single layer of wire are possible. The cover of the cavity was of glass 12 mm thick, and its sides were formed by rigid Teflon spacers with 1.5 mm of thickness. At the central position of two opposite edges of the cavity, the corresponding two Teflon spacers had straight channels with an aperture of 1.5 mm used to the forced injection of the wire. In order to reduce the friction, all parts of the cavity

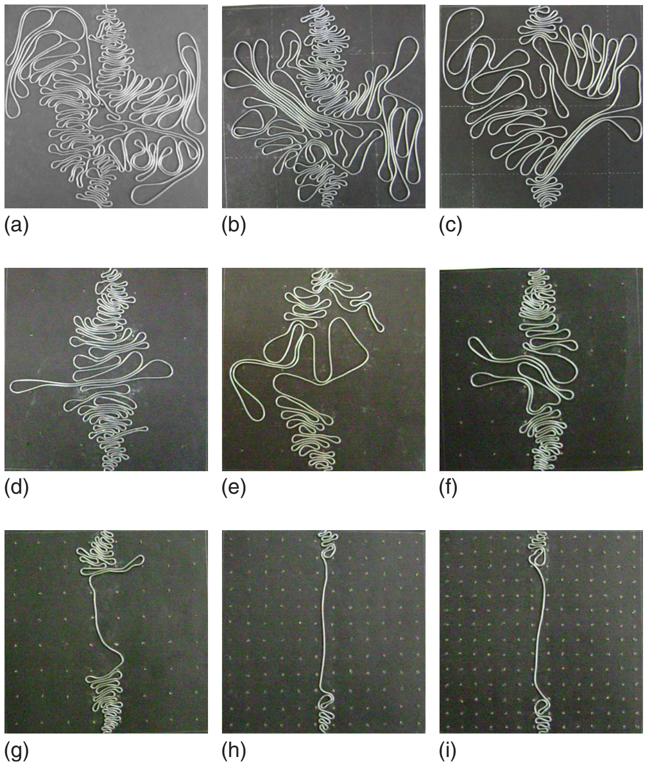

FIG. 1. (Color online) Typical morphologies of configurations of a $Pb_{0.40}Sn_{0.60}$ plastic wire with 1.5 mm of diameter injected in a quasi-2D cavity of 150^2 mm^2 of area and height 1.5 mm at the jamming limit, and with the following number n of pins localized on a square lattice: (a) $n=1$, (b) $n=4$, (c) $n=9$, (d) $n=16$, (e) $n=25$, (f) $n=36$, (g) $n=64$, (h) $n=144$, and (i) $n=196$. See text (Sec. II) for details.

were polished. Cavity and wire operated in dry regime, free of any lubricant. The plastic wires used were of the alloy $Pb_{0.40}Sn_{0.60}$ commonly used in welding in electronic devices.

Each injection experiment begins fitting a straight wire in the opposite channels and subsequently pushing manually and uniformly the wire on both sides of the cell toward the interior of the cavity. The injection velocity at each channel in these experiments was of the order of 1cm/s. For wires with the largest lengths, the crumpled structures formed become rigid, the difficulty in the injection increases very rapidly, and the injection velocity goes to zero with the formation of a jammed state of crumpled wire within the cavity. However, the observed phenomena are widely independent of the injection speed for all interval of injection velocity compatible with a manual process. Figures 1(a)–1(i) illustrate the morphologies of configurations of plastic 2D CW obtained at the jamming limit of maximum packing fraction for the following number of pins: (a) $n=1$, (b) $n=4$, (c) $n=9$, (d) $n=16$, (e) $n=25$, (f) $n=36$, (g) $n=64$, (h) $n=144$, and (i) $n=196$. The length L of the wire in the interior of the cavity in the jammed state is associated with the corresponding 2D packing fraction p through the relation $p=\zeta L/L_0^2$. Thus, the packing fraction at the beginning of each experiment is $p=p_{minimum}=\zeta L_0/L_0^2=\zeta/L_0$. Ten equivalent configurations of 2D-CW were used in this work for each cavity with a fixed number of pins. In average, the maximum [minimum] packing fraction found in the experiment (Fig. 1(a) [Fig. 1(i)]) was 0.32 [0.022] corresponding in average to a length L of plastic wire introduced in the cavity of 4800 mm [330 mm].

As shown in Fig. 1, the loop is the basic unit present in the nonthermal jammed state of the packing of plastic 2D CW studied in the present paper. Here a loop is defined as a portion of the wire formed by a bulge and a tail in two opposite extremities. The direction of a tangent vector moving on any loop varies continuously, with the exception that at the end of the two branches of each tail that vector points in opposite directions. Now, it is interesting to compare some basic and qualitatively new aspects of the plastic, irreversible, 2D-CW exemplified in Fig. 1 with the more elastic and less irreversible structures of crumpled wires previously studied [6,7]. First, each loop in the plastic case is individually less regular or less symmetric than in the elastic case. Second, the plastic overall structures are more heterogeneous and a cascade of hierarchical loops is less evident. Third, the plastic structure presents more loose domains, with low density of wire-wire contacts, particularly for low n. Differently of the crumpled state of elastic wires, here we observe frequently the occurrence of dangling loops, as clearly manifested in Figs. 1(a)–1(g). However, and in spite of these low-density domains, the global packing fraction of the configurations of plastic CW are larger than in the less irreversible case. This is due to the fact that elastic wires resist more efficiently to confinement. Fourthly, there is a tendency for the wire to distribute itself in some few regions of the cavity adopting columnar structures as a stack of lamellae, with some orientational order, and reminiscent of the structure of smectic liquid crystals. In particular, for n near or larger than 16, it is observed that a single stack of loops tends to be formed along the injection channels.

III. DISCUSSION

With the increase in the number of pins, n, the geometric hindrance factors increase as well and we expect that the packing fraction of the wire, $p(n)$, and the number of loops, $N(n)$, be decreasing functions of n. A precise and definitive theoretical explanation for the nonthermal packing problem of a wire in a 2D cavity with pins is out of the scope of this paper. However, we discuss here and in the next section heuristic mean-field arguments trying to describe and to understand the behavior of $N(n)$ and $p(n)$. First, we illustrate in Fig. 2, the dependence of the number of loops with n: $N(n)$ is a decreasing function of n for $n>1$ with slight fluctuations. The log-log plot in the inset of this figure shows that the number of loops exhibits a power-law tail over one decade for $n\geq 16$, with $N\sim n^{-1/2}$, within statistical uncertainties of less than 10% in the critical exponent. As shown in Fig. 1, we see that for $n\geq 16$, the loops tend to be approximately distributed along a one-dimensional single column geometry and the maximum number of loops must satisfy $N<N_{max}=L_0/\zeta=100$.

From Fig. 1 we observe that as the number of pins grows the dispersion in the size of the loops decreases. In particular, for $n\geq 16$ the size of the loops seems strongly controlled by

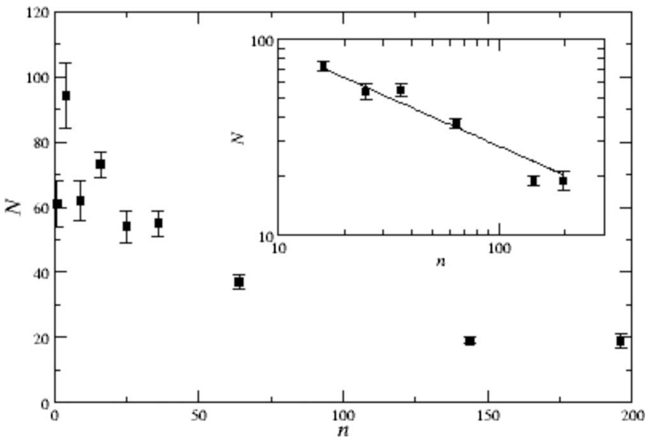

FIG. 2. The number of loops (and statistical fluctuations) as a function of the number of pins, $N(n)$, for plastic crumpled wire undergoing the process illustrated in Fig. 1. The values $N(n)$ are averages on ten similar experiments. The inset shows in log-log scales the power-law decay in $N(n)$ for $n \geq 16$, and the continuous line represents the best fit to the data $N \sim n^{-1/2}$. See text (Secs. II and III) for details.

the distance l between pins. From now on we assume that this size is of the order of l for $n \geq 16$. If we use the hypothesis that the number of loops for $n \geq 16$ scales with the area $A = L_0 l$ of the columnar region where the loops tend to concentrate, we get $N \sim L_0 l \sim n^{-1/2}$ which coincides with the asymptotic behavior of $N(n)$ shown in the inset of Fig. 2.

The dependence on the packing fraction with the number of pins is shown in detail in Fig. 3. The function $p(n)$ also exhibits a power-law tail in the same domain of the variable n, as observed for $N(n)$. In particular, the experimental data suggest that $p \sim n^{-\beta}$ with $\beta = 0.88 \pm 0.06$, from $n = 16$ to $n = 196$, i.e., in the regime dominated by the presence of pins and by a columnar array of loops. In principle, the packing fraction expected in this regime is approximately proportional to N times the average length of the loop ($\approx l$) or more precisely, $p \approx N l \zeta / {L_0}^2 \sim n^{-1/2}\, n^{-1/2} = n^{-1}$. This last estimate is, in fact, not too different from the scaling $p \sim n^{-0.88 \pm 0.06}$ obtained in the best fit shown in Fig. 3. For low number of pins ($n \leq 16$) the crumpled wire reaches the boundary of the cell with a distribution of heterogeneous loops that tend to spread throughout the cavity. In this region dominated by finite-size effects, there is an effective dependence $p \sim n^{-\alpha}$, with $\alpha = 0.23 \pm 0.03$, from $n = 1$ to $n = 16$.

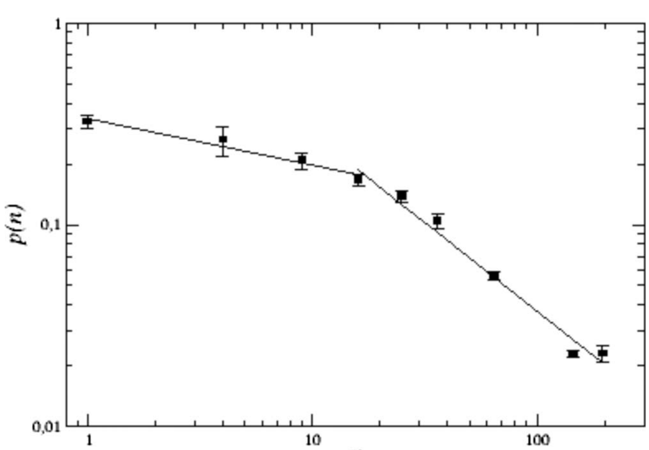

FIG. 3. Log-log plot of the experimental packing fraction $p(n)$ defined in Sec. II for a plastic wire at the jamming limit as a function of the number of pins n. The values $p(n)$ are averages on 10 similar experiments. The plot shows that $p(n)$ decays as a scaling law for $16 \leq n \leq 196$ with an exponent 0.88 ± 0.06. For $n < 16$ the packing is dominated by finite-size effects. See text (Secs. III and IV) for details.

The reader can observe in connection with Fig. 3 that the jammed average packing fractions associated with odd values of n (1, 9, and 25) are localized on the same trend lines of those corresponding to even values of n (4, 16, 36, 64, 144, and 196). This is interesting because if n is odd, there is a row of $m = n^{1/2}$ pins just along the axis connecting the two injection channels, and as a consequence there are 2^m different ways to dispose initially the wire, a situation completely different from n even. In this last case, there are no pins along the injection axis, and consequently there is only a single initial condition, namely the straight wire along the injection channels. The experiments show that the maximum packing fraction for n odd is independent of the initial condition of the wire. Furthermore, we can observe from the microscopic (local) point of view that the central array of pins in Figs. 1(c) and 1(e), corresponding, respectively, to the odd values $n = 9$ and $n = 25$, disturbs severely the distribution of loops within the cavity, especially if we compare the morphology observed for these two cases with those observed for the next configurations associated with the even values $n = 16$, and $n = 36$.

In principle, we can expect that there is a critical number of pins n_c such that $p(n_c) = p_{minimum} = \zeta / L_0$, that is a number of pins such that for $n \geq n_c$ the only configurations of the wire which are allowed are linear or approximately linear configurations along the injection axis. To find a theoretical upper bound n^* to n_c, we can consider a square lattice of pins whose distance between centers of nearest-neighbor pins is 3ζ. For this particular distribution of pins, the free spacing between the surfaces of nearest-neighbor pins is just 2ζ. In this particular case, we get $n^* = (L_0 / 3\zeta)^2$, and the wire rigorously cannot bend due to the lack of space. In practice, $n_c \ll n^*$: in our experiment, the value for n^* is approximately 1110, although extrapolation from Fig. 3 suggests that n_c is close to 400 pins (i.e., 1 pin for each 56 mm^2, corresponding to 3.14% of the total area of the cavity occupied by pins.). The packing fraction reaches the minimum value $p_{minimum} = 0.01$ in our experiment, while $p_{minimum} \to 0$ at the thermodynamic limit.

We conjecture that the phenomenon reported in the present paper, and the corresponding statistical functions discussed in the last paragraphs are essentially independent of the details of the geometry of the distribution of pins within the cavity, as well as of the symmetry of the cavity. As a support for this conjecture, Fig. 4 shows the experimental packing fraction $p(n)$ obtained with the same type of plastic wire, but for an off-lattice nondisordered distribution of pins following diverse geometric radial patterns in circular cavities of 150 mm of diameter. In this case, the number of pins within the cavity was 1, 2, 4, 9, 25, 64, 81, 100, and 169. As

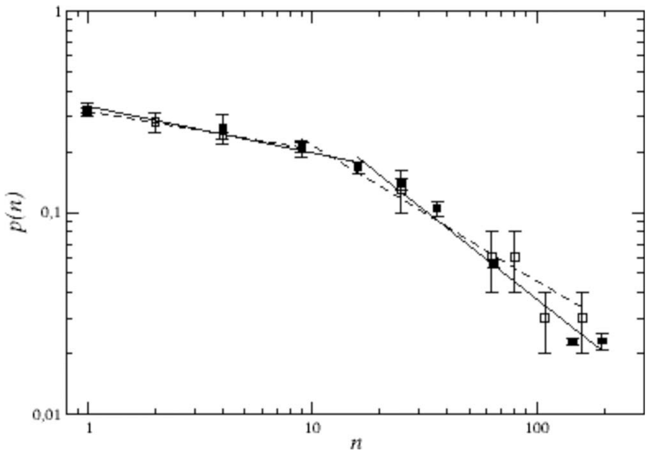

FIG. 4. Experimental packing density $p(n)$ for off-lattice nondisordered radial distributions of pins in circular cavities (dashed lines). The power-law fit has a scaling exponent $\beta=0.75\pm0.08$ for $25\leq n\leq 196$. For comparison, it is repeated in this figure the experimental data of Fig. 3 for square lattices of pins (power-law fit represented by continuous line).

in Fig. 3, the experimental data in Fig. 4 exhibit a power law (dashed line) with a scaling exponents $\beta=0.75\pm0.08$, for $16\leq n\leq 169$. For low number of pins, $n<16$, we have configurations which are limited by the finite size of the cavity, and an approximate dependence (dashed line) $p\sim n^{-\alpha}$, with $\alpha=0.20\pm0.02$ is still observed. In order to help the reader, we repeat for comparison in Fig. 4 the experimental data shown in Fig. 3 for the square lattice of pins (fits represented by continuous lines). Furthermore, another series of experiments using more elastic wires of copper confined in cavities containing distribution of pins on square lattices present a similar scaling law $p\sim n^{-\beta}$, with the exponent $\beta=1.00\pm0.10$, valid from $n=25$ to $n=196$. The region dominated by finite-size effects $(1<n<25)$ exhibits an approximate dependence $p\sim n^{-\alpha}$, with $\alpha=0.22\pm0.04$. In our overall estimate, irrespective the distribution of pins, the material of the wire, and the shape of the cavity, the value of the exponent β for the proposed scaling function $p(n)$ is 0.90 ± 0.10.

IV. MEAN-FIELD MODEL

In this section we discuss our experimental findings in terms of statistical thermodynamics ideas, exploring formal analogies with some basic results. First, we observe from Figs. 1 and 3 that the packing fraction $p=\zeta L/L_0^2$ tends to $p_{minimum}=\zeta L_0/L_0^2=\zeta/L_0$ (=0.01 in our experiment) for very large number of pins n: Fig. 1(g) indicates $p=0.055$, for $n=64$, and Figs. 1(h) and 1(i) indicate $p\approx 0.02$, for $n=144$ and 196, respectively. Physically, we expect that the average number of configurations $\Omega(p)$ accessible to a wire in our nonthermal experiment for a fixed p, satisfies $\Omega(p)\to 1$ as $p\to p_{minimum}$. Thus, the entropy $S(p)$ for a wire within the cavity is expected to satisfy $S(p)\sim \ln\Omega(p)\to 0$ close to the minimum value of p, which corresponds to the case of a linearly stretched wire along the injection channels. On the other hand, if the number of pins decreases the constraints to the motion of the wire within the cavity decrease as well, more equivalent configurations are accessible to the wire, and we expect that the entropy increases as a consequence. This reasoning suggests that we can associate to the nonthermal system studied here an effective temperature T which is a decreasing function of n. It must be emphasized that in the nonthermal crumpling of a wire in a cavity with pins the fluctuations are induced by the driving force of injection and do not arise from random thermal motion. Thus, for simply connected cavities free of pins we get the high (infinite) temperature limit of the system, for an intermediary number of pins (as exemplified in Figs. 2 and 3 by the interval $16<n<196$) we get the low-temperature limit of the system, and for very high number of pins we have the zero-temperature limit. We note that recent works have proposed that nonequilibrium systems experiencing jamming or structural arrest could be described by equilibrium thermodynamic concepts and an effective disorder temperature could be introduced to characterize the properties of such systems [24–26].

Using simple heuristic mean-field arguments we now introduce an effective internal energy E for a wire confined in a cavity with pins and after we construct an effective Helmholtz free energy $F=E-TS$ specially valid for the low-effective T limit (equivalently, the intermediary number of pins limit, $16<n<196$, or low packing density limit). As a first hypothesis, for E we adopt a repulsive energy due to self-avoidance effects between different parts of the crumpled wire as long as we can neglect the elastic energy of curvature for a plastic wire. As the packing fraction p is a measure of the mean local concentration of matter, the average repulsive energy per unit volume is proportional to the number of pairs of interacting pieces of the wire, i.e., to p^2. For the entire cavity with a fixed volume $V=L_0^2\zeta$ we get for the repulsive energy $E=\varepsilon.p^2$, with ε being a constant. Finally, as a second hypothesis we assume for the same region $16<n<196$ an entropy S scaling linearly with the total length of the wire, that is $S\sim L\sim p$, or $S=\sigma.p$, with σ constant. Consequently, $F=E-TS=\varepsilon.p^2-T\sigma.p$. In the minimization of F, when p varies, the first term, E, due to self-avoidance repulsion, favors small values of p or L, while the second term due to entropy, $-TS$, tend to favor large values of p or L. Thus, $\partial F/\partial p=0$ leads to the equilibrium condition for the effective temperature $T=(2\varepsilon.p/\sigma)=(\partial E/\partial S)_V$, with $p=p(n)$. If we identify $p(n)$ in the last equation with our overall estimate for the jammed packing fraction given at the end of the previous section, namely $p\sim n^{-0.90\pm0.10}$, valid in the interval $16<n<196$, we obtain as physically expected and discussed in the previous paragraph an effective temperature that decreases with the number of pins, $T\sim p\sim n^{-(0.90\pm0.10)}$. As a consequence, in the present analogy there is a connection of the effective temperature T with the topology of the system (given by the number of pins n). The effective temperature T measures the disorder of the wire in the cavity: as long as the number of pins increases, the average heterogeneity tends to decrease and more regular structures appear as we can convince ourselves from an exam in the typical configurations shown in Fig. 1.

It is interesting to observe that the arguments discussed above imply that the "equilibrium" value for the internal energy of the plastic crumpled wire will be the quadratic function of the effective temperature given by: $E=\varepsilon . p^2 =(\sigma^2/4\varepsilon)T^2$. This result is analogous to the temperature dependent energy of a Fermi gas with two-body interactions at low temperature as given by the Hartree-Fock theory [27]. From the last result we obtain that the corresponding effective specific heat of the system is given by $C_V=(\partial E/\partial T)_V =(\sigma^2/2\varepsilon)T=\sigma . p=S$. According to the analogy discussed here, the plots of $p(n)$ in Figs. 3 and 4 represent essentially a constant times the entropy or the effective specific heat of the system. Furthermore, the number of loops in the "low-temperature region" considered, $16<n<196$, which scales as $N\sim n^{-1/2}$ (Fig. 2), will correspond approximately to a scaling law with the square root of the effective temperature, i.e., $N\sim T^{1/2}$.

V. CONCLUSION

A new packing problem in 2D is studied experimentally in the present paper. The experiments examine in detail the crumpled structures formed when a plastic wire is injected into 2D square cavities of area $L_0{}^2=150^2$ mm^2 and with different topologies, given by different distributions of pins localized on square lattices. The diameter of the pins and of the plastic wire, as well as the height of the pins and cavities, has the same measure $\zeta=L_0/100=1.5$ mm in such a way that only structures with a single layer of wire are possible (Fig. 1). Besides its intrinsic interest, the present study can be related to a number of important fields including self-avoiding walks and polymer configurations on the plane, packing of DNA segments constrained to wrap around histone cores [22], and the electrohydrodynamics of DNA in confined environments [23]. In spite of irreversibility and the emergence of several types of geometric motifs, robust scaling laws are observed for the number of loops and for the jammed packing fraction of the plastic wire as a function of the number of pins within the cavity (Figs. 2–4). Surprisingly, extensive experimental results suggest that the critical exponents associated with the observed scaling laws are largely independent of the details of the distribution of pins within the cavity, as well as independent of the symmetry of the cavity, and the type of the wire. Simple mean-field arguments suggest a relation between the number of pins within the cavity and an effective absolute temperature for this nonthermal system.

ACKNOWLEDGMENTS

We acknowledge CAPES, CNPq, FAPEPI, PRONEX, and UFPI for financial support. The authors are grateful to M. C. Vieira for helpful comments and to E. J. R. Parteli for a critical reading on the manuscript. Valuable comments from an anonymous referee are acknowledged.

[1] M. N. Barber, Phys. Rep. **59**, 375 (1980).
[2] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[3] K. J. Strandburg, Rev. Mod. Phys. **60**, 161 (1988).
[4] S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature (London) **393**, 550 (1998).
[5] R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, and A. P. Mackenzie, Science **315**, 214 (2007).
[6] C. C. Donato, M. A. F. Gomes, and R. E. de Souza, Phys. Rev. E **66**, 015102(R) (2002).
[7] C. C. Donato, M. A. F. Gomes, and R. E. de Souza, Phys. Rev. E **67**, 026110 (2003).
[8] C. C. Donato, F. A. Oliveira, and M. A. F. Gomes, Physica A **368**, 1 (2006).
[9] L. Boué, M. Adda-Bedia, A. Boudaoud, D. Cassani, Y. Couder, A. Eddi, and M. Trejo, Phys. Rev. Lett. **97**, 166104 (2006).
[10] L. Boué and E. Katzav, EPL **80**, 54002 (2007).
[11] M. Habibi, N. M. Ribe, and D. Bonn, Phys. Rev. Lett. **99**, 154302 (2007).
[12] C. C. Donato and M. A. F. Gomes, Phys. Rev. E **75**, 066113 (2007).
[13] V. Romero, T. A. Witten, and E. Cerda, Proc. R. Soc. London, Ser. A **464**, 2847 (2008).
[14] E. Katzav, M. Adda-Bedia, and A. Boudaoud, Proc. Natl. Acad. Sci. U.S.A. **103**, 18900 (2006).
[15] M. A. F. Gomes, V. P. Brito, and M. S. Araújo, J. Braz. Chem. Soc. **19**, 293 (2008).
[16] N. Stoop, F. K. Wittel, and H. J. Herrmann, Phys. Rev. Lett. **101**, 094101 (2008).
[17] Y. C. Lin, Y. W. Lin, and T. M. Hong, Phys. Rev. E **78**, 067101 (2008).
[18] M. A. F. Gomes, V. P. Brito, A. S. O. Coelho, and C. C. Donato, J. Phys. D **41**, 235408 (2008).
[19] A. S. Balankin, D. M. Matamoros, E. P. León, A. H. Rangel, M. A. M. Cruz, and D. S. Ochoa, Physica A **388**, 1780 (2009).
[20] B. Carneiro da Cunha, EPL **88**, 31001 (2009).
[21] P.-G. de Gennes, *Scaling Concepts in Polymer Physics* (Cornell University Press, Ithaca, 1979).
[22] C. Vaillant, B. Audit, and A. Arnéodo, Phys. Rev. Lett. **95**, 068101 (2005).
[23] O. B. Bakajin, T. A. J. Duke, C. F. Chou, S. S. Chan, R. H. Austin, and E. C. Cox, Phys. Rev. Lett. **80**, 2737 (1998).
[24] H. A. Makse, Physica A **330**, 83 (2003).
[25] N. Xu and C. S. O'Hern, Phys. Rev. Lett. **94**, 055701 (2005).
[26] E. Bouchbinder, Phys. Rev. E **77**, 051505 (2008).
[27] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 2003).